LIFT CONTROL IN MAGNETOHYDRODYNAMICS *

(REGULIROVANIE POD"EMNOI SILY V MAGNITNOI GIDRODINAMIKE)

PMM Vol.28, № 2, 1964, pp. 268-279

Adolf BUSEMANN

(Langley Field, Va., U.S.A.)

(Received October 2, 1963)

In the first paper for the Hydrodynamical Section of this Symposium about Analytic Functions in Continuum Mechanics I like to return to an early and famous application in classical hydrodynamics made by W.M. Kutta and N.E. Zhukovskii while studying the lift force created in two-dimensional flow by a perfect fluid. The doubly connected region around a single profile in the infinite plane allows for a circulatory motion, the intensity of which is not determined by immediate boundary conditions. Nevertheless both these authors selected the same condition of smooth flow at the sharp trailing edge of either the flat plate (Kutta) or the special series of sharply edged Zhukovskii profiles as the proper lift-controlling element for the perfect flow, generally known as the "Kutta-Zhukovskii Condition". The smooth flow, however plausible it may appear, can only be a conjecture within the theory of perfect fluids and the rigorous proof resulting from flow problems with vanishing viscosity was added soon after by the boundary-layer theory of L. Prandtl.

A similar problem is found in the modern magnetohydrodynamics of two dimensions with a fluid having two properties to perfection, that is, vanishing viscosity paired with vanishing electrical resistivity while an external magnetic field is spead throughout the whole flow field. Regarding the distribution of two fields, the velocity and the magnetic vectors, it may appear that there are also two circulations around the profile undermined, but, as is easy to conceive, these two circulations are fortunately tied to each other and only one combined circulation remains open and is responsible for any side force or lift. Unfortunately the simplified treatment adopted in this paper is a linearized approach to any lift-controlling elements of the body shape. Therefore, the ease to find a large variety of illustrative examples must be relied upon to hope that the linearized method may illuminate the problem sufficiently to disclose the magnetohydrodynamical generalization of the Kutta-Zhukovskii as a similarly plausible conjecture.

In this paper, a short derivation of the differential equations and finally their specialization to two-dimensional flow will be given. Immediately thereafter, the general solution will be produced just as in my von Kármán 80th Anniversary Lecture, 1961. It is exactly this step that led me to believe that its discussion at this Symposium might be appropriate. After showing a couple of examples in figures and pointing out their essential features, some conclusions about the lift-controlling elements will be added.

---

* Presented at the International Symposium on Application of the Theory of Functions in Continuum Mechanics. Tbilisi, September 17 – 23, 1963.

1. Differential equations. For the sole purpose of showing the changes of the Kutta-Zhukovskii lift problem by electromagnetic interaction, it is permissible to take the name magnetohydrodynamics literally by assuming an incompressible fluid with a constant electric conductivity value regarded as a scalar quantity. The essential and, in the end, the only remaining variables of the flow field are the two vectors: the magnetic induction $\mathbf{B}$ and the fluid velocity $\mathbf{V}$. Both these vector fields have lost part of their generality by the absence of divergences

$$\operatorname{div} \mathbf{B} = 0 \tag{1.1}$$

$$\operatorname{div} \mathbf{V} = 0 \tag{1.2}$$

the first equation being a natural property by selecting the magnetic inductions as the representative of the magnetic field strength, the second equation being a mere convenience resulting from the assumption of incompressible fluid.

The square of both field vectors $\frac{1}{2\mu}B^2$ and $\frac{1}{2}\rho V^2$ indicate in connection with the proper constants in a magnetically permeable and incompressible fluid magnetic pressures and hydrodynamic pressures. This property makes them commensurable in the form of the Alfven speed $B/\sqrt{\rho\mu}$ compared with the velocity V. To simplify the equations, the density ρ and the permeablity μ are set equal to unity. Thus, the kinematic viscosity is the proper dissipative constant in this system, while for the kinematic "resistivity" of electrical currents, the reciprocal conductivity has to be divided by the permeability μ

$$k = \frac{1}{\sigma\mu} \tag{1.3}$$

The influence on the hydrodynamic behavior of the movement across a magnetic field is caused by the so-called Lorentz forces of electrodynamics and their value $\mathbf{f}$ per unit volume is dependent on the flow of an electric current with the area intensity $\mathbf{j}$

$$\mathbf{f} = \mathbf{j} \times \mathbf{B} \tag{1.4}$$

however, any current reveals itself by the curling magnetic field around it according to Equations

$$\mathbf{j} = \operatorname{curl} \mathbf{B} \tag{1.5}$$

In combining these two expressions, the L rentz forces immediately show how they are anchored in the magnetic field stress tensor $B_iB_k - {}^1/_2\, \delta_{ik}B^2$

$$\mathbf{f} = (\operatorname{curl} \mathbf{B}) \times \mathbf{B} = \mathbf{B} \cdot \nabla \mathbf{B} - {}^1/_2 \nabla \mathbf{B}^2 \tag{1.6}$$

These forces enter the hydrodynamic equation

$$\mathbf{f} - \operatorname{grad} p_h = \frac{\partial V}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} - \nu \nabla^2 \mathbf{V} \tag{1.7}$$

The magnetic pressure p_m, the gradient of which is visible in the last term of Equations (1.6), offers a summation with the static pressure p_h to the combined pressure p as indicated

$$p = p_h + p_m = p_h + {}^1/_2 B^2 \tag{1.8}$$

the gradient of which is simply

$$-\operatorname{grad} p = \left(\frac{\partial}{\partial t} + \mathbf{V}\cdot\nabla - \nu\,\nabla^2\right)\mathbf{V} - (\mathbf{B}\cdot\nabla)\,\mathbf{B} \tag{1.9}$$

In the absence of imposed electric fields all currents $\mathbf{j}$ are caused by induction in the fluid moving across the magnetic field or vice versa. Closed currents are already assured when firstly the curl of the inducted electric field $\mathbf{E}_{in}$ is used*

$$\operatorname{curl}\mathbf{E}_{in} = \operatorname{curl}(\mathbf{V}\times\mathbf{B}) - \frac{\partial\mathbf{B}}{\partial t} \tag{1.10}$$

and secondly the electric resistivity $\varkappa$ of the fluid is a constant

$$\operatorname{curl}\mathbf{E}_{in} = \varkappa\operatorname{curl}\mathbf{j} = \operatorname{curl}(\mathbf{V}\times\mathbf{B}) - \frac{\partial\mathbf{B}}{\partial t} \tag{1.11}$$

Avoiding again the explicit appearance of the currents or the electric field, a relation between the magnetic field and the velocity field results out of Equation (1.11) combined with (1.5)

$$\varkappa\operatorname{curl}\operatorname{curl}\mathbf{B} = \operatorname{curl}(\mathbf{V}\times\mathbf{B}) - \frac{\partial\mathbf{B}}{\partial t} \tag{1.12}$$

After the usual transformation of all double cross products into two terms of dot products, while considering that all divergences vanish according to Equations (1.1) and (1.2), the same relation can be written

$$-\varkappa\,\nabla^2\mathbf{B} = \mathbf{B}\cdot\nabla\mathbf{V} - \mathbf{V}\cdot\nabla\mathbf{B} - \frac{\partial\mathbf{B}}{\partial t} \tag{1.13}$$

All differential operations except one are carried out on the magnetic field. Two operator packages in parentheses show this fact more clearly

$$\left(\frac{\partial}{\partial t} + \mathbf{V}\cdot\nabla - \varkappa\,\nabla^2\right)\mathbf{B} - (\mathbf{B}\cdot\nabla)\,\mathbf{V} = 0 \tag{1.14}$$

This final form of electromagnetic equation has a strange resemblance to the final hydrodynamic equation (1.9). The third terms of the large operators differ only by their diffusion coefficients which are the viscosity ν in hydrodynamics and the resistivity $\varkappa$ in electronagnetism. If such a difference, which of course disappears for perfect fluids, should cause too much trouble, there is always the possibility of finding a first solution for the fluid with $\nu = \varkappa$ or the "equi-dissipative" fluid.

2. Linearisation. Any differential operator parenthesis used in the final equations (1.9) and (1.14) contains a term having one of the unknown field vectors as a coefficient, as a reminder of the nonlinearity of both relations. In these terms lies the actual difficulty of solving the complete

---

* The invariant field E_{in} is related to the Cartesian E_{ca} as follows:

$$E_{in} = E_{ca} + V \times B$$

problem by finding particular solutions. However, such an approach can be used after linearization of the differential equations for small disturbance flows. Since all terms in the operators represent partial differentiations with respect to space or time, an undisturbed pattern consisting of a parallel flow and a parallel magnetic field – though in different directions, if preferred – vanishes identically if operated upon regardless of any unknown field used inside the operators. Under these circumstances the linearization requires the unknown disturbance fields to be always the object of the operations, while the field inside the operators may as well be the unadulterated undisturbed one.

If we designate with the former letters $\mathbf{V}$ and $\mathbf{B}$ the main fields and use for the disturbances $\mathbf{v}$ and $\mathbf{b}$ as small additions to the parallel fields, the operators in equations (1.9) and (1.14) are practically differential operators with fixed coefficients in which the field vectors $\mathbf{V}$ and $\mathbf{B}$ are considered to be the values $\mathbf{V}_\infty$ and $\mathbf{B}_\infty$ sufficiently far away from the disturbing body, without being explicitly designated as such by a subscript. This linearization allows the operators to be treated as commuting factors as long as they operate on the same field $\mathbf{v}$ or $\mathbf{b}$. The new form of Equations (1.9) and (1.14) due to linearization is

$$\left(\frac{\partial}{\partial t} + \mathbf{V}\cdot\nabla - \nu\nabla^2\right)\mathbf{v} - (\mathbf{B}\cdot\nabla)\,\mathbf{b} = -\operatorname{grad} p \tag{2.1}$$

$$\left(\frac{\partial}{\partial t} + \mathbf{V}\cdot\nabla - \varkappa\,\nabla^2\right)\mathbf{b} - (\mathbf{B}\cdot\nabla)\,\mathbf{v} = 0 \tag{2.2}$$

The early equations (1.1) and (1.2) adapted to the new symbols for the disturbance fields are

$$\operatorname{div}\mathbf{b} = 0, \qquad \operatorname{div}\mathbf{v} = 0 \tag{2.3}$$

It is obvious that these four equations, half of which are scalar equations while the other half are vectors with the disturbance function p, have enough information to allow the unknown fields to be determined.

3. Reduction to two dimensions. Since the best hope of finding solutions is given in the two-dimensional flow, the further treatment of these equations is carried out in the complex plane in accord with Kutta and Zhukovskii. The first convenience resulting from the reduction to two dimensions is the existence of a scalar flux function φ for the magnetic field $\mathbf{b}$ and a scalar stream function ψ for the velocity field as integrals of Equations (2.3). In complex plane $z = x + iy$ with $\bar{z} = x - iy$ the nabla operator is known to be

$$\nabla = 2\,\frac{\partial}{\partial \bar{z}}$$

of any function written in the variables z and $\bar{z}$. While a gradient can immediately be written in nabla, the relation between flux functions or stream functions and the vector field parallel to them only requires a multiplier $\pm i$ to perform a rotation about $\pm 90°$. The vector fields are, therefore, easily related to their flux integrals

$$\mathbf{b} = -i2\frac{\partial}{\partial \bar{z}}\varphi, \qquad \mathbf{v} = -i2\frac{\partial}{\partial \bar{z}}\psi \tag{3.1}$$

While Equation (2.2) may be raised to the flux function level

$$\left(\frac{\partial}{\partial t} + \mathbf{V}\cdot\nabla - \varkappa\nabla^2\right)\varphi - (\mathbf{B}\cdot\nabla)\psi = 0 \tag{3.2}$$

the right-hand side of Equation (2.1) is not quite suited to this treatment. But the vanishing divergences of $\mathbf{v}$ and $\mathbf{b}$ spotted on the left-hand side disclose p to be an analytic function in the whole domain outside the body

$$\text{div grad } p = 0 \tag{3.3}$$

If this fact is appreciated, the pressure p must be real part of a complex "pressure" function π

$$p + iq = \pi(z, \bar{z}, t) \tag{3.4}$$

Since p and its complex conjugate q carry the same information, it is simple to conceive that i grad q can replace grad p as a result of conformal mapping. The raising of Equation (2.1) to flux level is consequently performed by exchange of p with q

$$\left(\frac{\partial}{\partial t} + \mathbf{V}\cdot\nabla - \nu\nabla^2\right)\psi - (\mathbf{B}\cdot\nabla)\varphi = q \tag{3.5}$$

with undestanding that q is an analytic function

$$\text{div grad } q = \nabla^2 q = 4\frac{\partial^2}{\partial z \partial \bar{z}} q = 0 \tag{3.6}$$

Equations (3.5) and (3.2) represent two scalar equations for two unknown scalar functions φ and ψ while q may already serve as an arbitrary analytic function encountered during the integration.

4. General solution. Equation (3.2) can be integrated by a "potential" Q because of the commuting of all differential operators used

$$\left(\frac{\partial}{\partial t} + \mathbf{V}\cdot\nabla - \varkappa\nabla^2\right)Q = \psi, \qquad (\mathbf{B}\cdot\nabla)\,Q = \varphi \tag{4.1}$$

and the new function $Q(z, z, t)$ gets its restriction from Equation (3.5)

$$\left[\left(\frac{\partial}{\partial t} + \mathbf{V}\cdot\nabla - \nu\nabla^2\right)\left(\frac{\partial}{\partial t} + \mathbf{V}\cdot\nabla - \varkappa\nabla^2\right) - (\mathbf{B}\cdot\nabla)^2\right]Q = q \tag{4.2}$$

While the second summand is always a square, the first summand would also be a square of a single operator in the case of perfect flow without any dissipations. If however dissipations are considered essential, only a fluid with equal dissipation coefficients $\varkappa = \nu$, the "equi-dissipative" fluid, allows to set

$$\left[\left(\frac{\partial}{\partial t} + \mathbf{V}\cdot\nabla - \nu\nabla^2\right)^2 - (\mathbf{B}\cdot\nabla)^2\right]Q = \left[\left(\frac{\partial}{\partial t} + (\mathbf{V} - \mathbf{B})\cdot\nabla - \nu\nabla^2\right)\times \right.$$
$$\left.\times\left(\frac{\partial}{\partial t} + (\mathbf{V} + \mathbf{B})\cdot\nabla - \nu\nabla^2\right)\right]Q = q \tag{4.3}$$

According to the character of the "conjugate to the pressure" q of equation (3.6), the new function Q may be written as the operand of four independent operators

$$\left(\frac{\partial}{\partial t} + (\mathbf{V}-\mathbf{B})\cdot\nabla - \nu\nabla^2\right)\left(\frac{\partial}{\partial t} + (\mathbf{V}+\mathbf{B})\cdot\nabla - \nu\nabla^2\right)\left(2\,\frac{\partial}{\partial z}\right)\left(2\,\frac{\partial}{\partial \bar{z}}\right) Q = 0 \tag{4.4}$$

It promises an integral of four additive contributions

$$Q = Q_1 + Q_2 + Q_3 + Q_4 \tag{4.5}$$

With a specific adaption to one operator in each part

$$\begin{aligned} &\left(\frac{\partial}{\partial t} + (\mathbf{V}-\mathbf{B})\cdot\nabla - \nu\nabla^2\right) Q_1 = 0, \qquad 2\,\frac{\partial}{\partial z}\, Q_3 = 0 \\ &\left(\frac{\partial}{\partial t} + (\mathbf{V}+\mathbf{B})\cdot\nabla - \nu\nabla^2\right) Q_2 = 0, \qquad 2\,\frac{\partial}{\partial \bar{z}}\, Q_4 = 0 \end{aligned} \tag{4.6}$$

Going back to the original unknowns is necessary for the boundary conditions. Equations (4.1) lead back one step to the flux and stream functions, while Equations (3.1) add the second step toward the magnetic fields and velocity fields themselves. The application of (4.1) on parts of Q discloses their specific meaning

$$\left[\frac{\partial}{\partial t} + (\mathbf{V}-\mathbf{B})\cdot\nabla - \nu\nabla^2\right] Q_1 = \psi_1 - \varphi_1 = 0 \tag{4.7}$$

$$\left[\frac{\partial}{\partial t} + (\mathbf{V}+\mathbf{B})\cdot\nabla - \nu\nabla^2\right] Q_2 = \psi_2 + \varphi_2 = 0 \tag{4.8}$$

Since the only disturbances possible in fields with vanishing divergences are vortices, the solution Q_1 indicates co-rotating vorticity of both velocity and magnetic field travelling with the speed $\mathbf{V} - \mathbf{B}$ except for dissipation. In the same manner Q_2 is a contribution by contra-rotating vorticity in both fields travelling with the speed $\mathbf{V} + \mathbf{B}$. When the steady flow state is reached, the vorticity along any line $\mathbf{V} - \mathbf{B}$ or $\mathbf{V} + \mathbf{B}$ is uniform except for dissipation and the flow pattern resembles very much the supersonic two-dimensional flow (Fig. 1). The time dependent terms in the above equations are very useful in indicating the directions in which to employ the two families of waves starting at the body contour. Playing with this portion of the solution reveals only (slender) bodies without angle of attack to have neither a sink nor a source of magnetic flux inside the body. No pressures are created and the lift is always zero. But this is only the timid approach to studying lift. Pressures result from the solution Q_3 and Q_4 representing complex analytical functions of $\bar{z}$ respectively of z.

While reducing Q_3 and Q_4 to the flux and stream functions accoring to Equations (4.1), it is always allowable to omit the last term in the operator of the first equation (4.1), since analytic functions ha no dissipative contributions

$$\varphi_3 = \mathbf{B}\cdot\nabla Q_3, \qquad \psi_3 = \left(\frac{\partial}{\partial t} + \mathbf{V}\cdot\nabla\right) Q_3 \tag{4.9}$$

$$\varphi_4 = \mathbf{B}\cdot\nabla Q_4, \qquad \psi_4 = \left(\frac{\partial}{\partial t} + \mathbf{V}\cdot\nabla\right) Q_4 \tag{4.10}$$

For the final steady flow state the relations between the flux and the stream disturbance are even closer without the first terms for the stream function

$$\psi_3 = \mathbf{V}\cdot\nabla Q_3 = \left(V \frac{\partial}{\partial z} + \overline{V}\frac{\partial}{\partial \bar{z}}\right) Q_3(\bar{z}) = \overline{V} Q_3' \quad (4.11)$$

$$\varphi_3 = \mathbf{B}\cdot\nabla Q_3 = \left(B \frac{\partial}{\partial z} + \overline{B}\frac{\partial}{\partial \bar{z}}\right) Q_3(\bar{z}) = \overline{B} Q_3' \quad (4.12)$$

In the steady state the complex potentials are simply proportional

$$\varphi_3 = \frac{\overline{B}}{\overline{V}}\psi_3 \quad (4.13)$$

The velocities derived from these potentials have the same ratio

$$b_{34} = \frac{\overline{B}}{\overline{V}} v_{34} \quad (4.14)$$

The singularities from these disturbances are normally vortices hidden inside the body and represent outside of the body the circulatory motion or

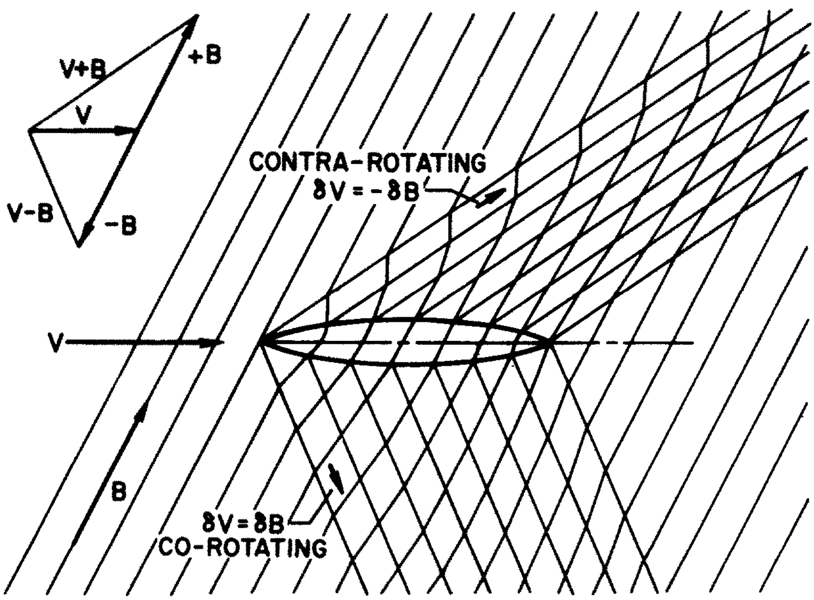

Fig. 1

magnetic field responsible for the lift. Both the flow and the magnetic field are not allowed to show sinks or sources, but a complex multiplier on one potential as in Equation (4.13) intermixes circulations and sources. The result shown in Fig. 2 is that the familiar circulation of the velocity field would change into a circulation plus a sink for the magnetic field.

If we reject the solutions Q_3 and Q_4 except for parallel fields V and B, because of the unaccountable sink strength, the timid approach to studying lift would be perfect. The real conclusion is to combine the flux surplus in any solution Q_1 and Q_2 created by finite angle of attack with the flux sink of Q_3 and Q_4 , and to get lift, or at least circulation controlled by the angle of attack. Fig. 3 and 4 are prepared to help in finding

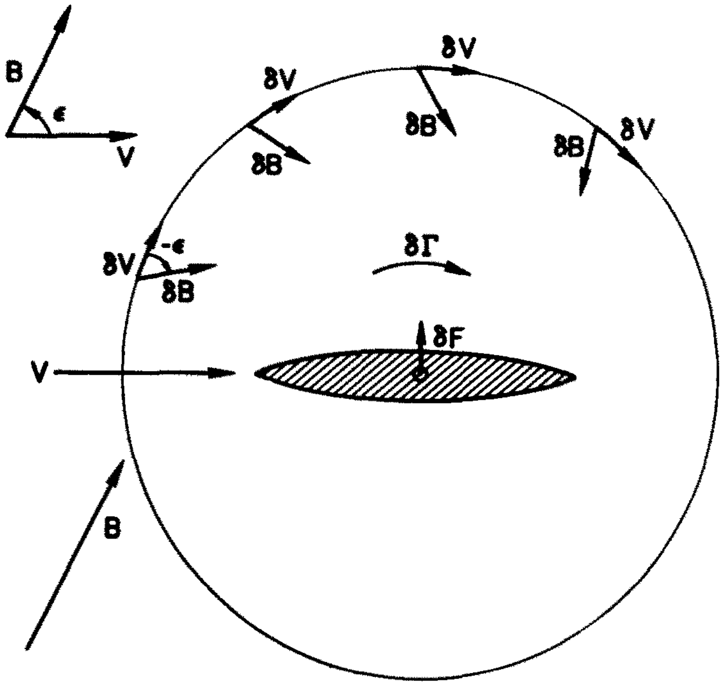

Fig. 2

the flux surplus $\Delta\varphi_{12}$ according to angle of attack. With the help of the fishbone pattern inside the body of Fig.1 in Fig.3, starting at the leading edge of the body, the bookkeeping of the magnetic flux for any body shape is simple. At zero angle of attack the surplus is zero, the horizontal to vertical size relations are taken from the vector diagram **V** , **B** to demonstrate that $2V$ horizontally corresponds to B_y vertically (Fig.4). The flux through any horizontal line unit is also B_y . The result for a chord c and an angle β downward amounts to

$$\Delta\varphi_{12} = cB_y\beta \frac{2V}{B_y} 2Vc\beta \tag{4.15}$$

With the same coordinates lined with x parallel to V the ratio $\bar{B}/\bar{V}$ is changing the velocity circulation Γ into the magnetic circulation J and the magnetic source strength S

$$J + iS = \frac{\overline{B}}{\overline{V}}\,\Gamma = \frac{B_x - iB_y}{\mathbf{V}}\,\Gamma \tag{4.16}$$

A source strength compensating the surplus of flux in Equation (4.15) leads to the circulation Γ

$$\Gamma = \frac{2V^2 c}{B_y}\beta \tag{4.17}$$

The force created by circulation in the combined hydromagnetic field is composed of the lifting force $iV\Gamma$ and the Lorentz force $-iBJ$. But instead of the expected simple current J indicating the magnetic circulation

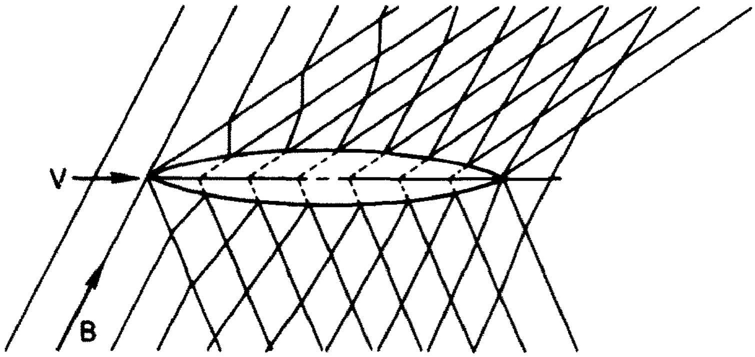

Fig. 3

around the body, the complex multiplication (4.15) creates $J = \Gamma\frac{\bar{B}}{\bar{V}}$. The result is

$$F = iV\Gamma - iB\left(\Gamma\frac{\bar{B}}{\bar{V}}\right) = iV\Gamma\left(1 - \frac{B\bar{B}}{V\bar{V}}\right) \tag{4.18}$$

The resulting force F is indeed a side force in the manner of the hydrodynamic lift, but it is reduced by the factor $1 - (B/V)^2$ which indicates vanishing lift at movements with Alfvén speed and a lift reversal for sub-Alfvénic speeds that is to say too strong magnetic fields.

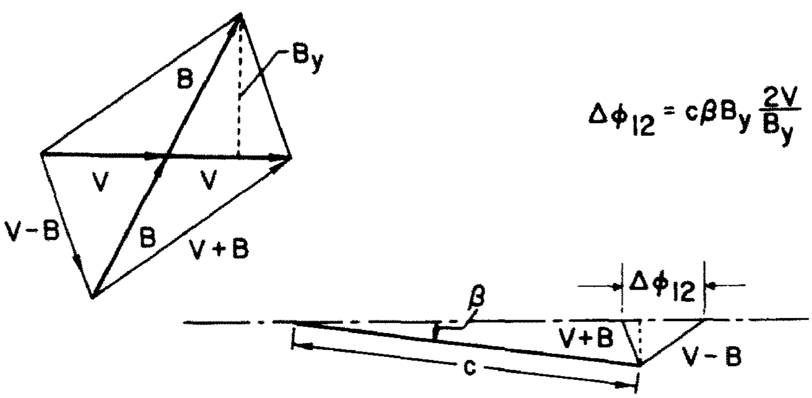

Fig. 4

Concerning the generalized Kutta-Zhukovskii condition, the case of non-alined fields V and B seems quite like supersonic flow, where the leading and the trailing edge may help to define the lift with precision by being sharpened, while the angle of attack is in complete control of the circulation.

5. Movement parallel to magnetic field. While the Kutta-Zhukovskii problem is seen to disappear as such, when the main magnetic field is under a finite angle with respect to the flow, the alined motion with (or against) the direction of the magnetic lines is an exception, in which the two vorticity strips starting at the body surface of Fig.1 degenerate to a single streamline behind or in front of the body. The potential flow disturbance according to the combined solutions Q_3 and Q_4 makes the magnetic lines coincide with the steady-state streamlines. Thus the problem in the large is exactly the familiar hydrodynamic problem and after it is solved, its streamlines are also used as magnetic lines. Only the vanishing imperfections by viscosity and by resistivity must be investigated for differences in the wake regions.

Mathematically the alinement of both the undisturbed velocity V and the magnetic field B with the x-axis is of not much immediate consequence. The general solution of Equation (4.3) still has too many different terms

$$\left[\left(\frac{\partial}{\partial t} + V\frac{\partial}{\partial x} - \nu\nabla^2\right)\left(\frac{\partial}{\partial t} + V\frac{\partial}{\partial x} - \varkappa\nabla^2\right) - B^2\frac{\partial^2}{\partial x^2}\right] Q = q \qquad (5.1)$$

If, however, the steady flow is studied with a semi-perfect fluid in which either the electric resistivity or the viscosity is zero, one factor $\frac{\partial}{\partial x}$ will be found separable

$$\left[\left(\frac{(V^2 - B^2)}{V}\frac{\partial}{\partial x} - \nu\nabla^2\right)\left(V\frac{\partial}{\partial x}\right)\right] Q = q \quad \text{for } \varkappa = 0 \qquad (5.2)$$

or

$$\left[\left(\frac{(V^2 - B^2)}{V}\frac{\partial}{\partial x} - \varkappa\nabla^2\right)\left(V\frac{\partial}{\partial x}\right)\right] Q = q \quad \text{for } \nu = 0 \qquad (5.3)$$

Splitting the first particular solutions Q_1 and Q_2 between these factorized operators as it is done for the equi-dissipative fluid, makes the second one Q_2 uninteresting, while the first one Q_1 has not $V - B$ but a new convective velocity $(V^2 - B^2)/V$ modifying the dissipation of vorticity.

This new convective velocity joins the zero value of $V - B$ but has the added factor $(V + B)/V$ which doubles for flows with about Alfvén speed and grows unlimitedly with increasing B. Having no immediate check on the non-steady build-up process, this new velocity resembles a "phase speed" compared to the "ground speed" $V - B$ of the equi-dissipative fluid.

Both extremes of semi-perfect fluids differ with respect to each other, when the flux function and the stream function are derived from Equations (4.1). For vanishing electrical resistivity (superconductivity) there results

$$\varphi_1 = B\frac{\partial}{\partial x}Q_1, \qquad \psi_1 = V\frac{\partial}{\partial x}Q_1 \quad \text{or} \quad \varphi_1 = \frac{B}{V}\psi_1 \quad \text{for} \quad \varkappa = 0 \tag{5.4}$$

while for nonviscous fluid it is found that

$$\left(\frac{(V^2-B^2)}{V}\frac{\partial}{\partial x} - \varkappa\nabla^2\right)Q_1 = \psi_1 - \frac{B}{V}\varphi_1 = 0 \tag{5.5}$$

or

$$\varphi_1 = \frac{V}{B}\psi_1 \quad \text{for } \nu = 0$$

The result is that the superconductive fluid conserves the plane similarity $\varphi : \psi = B : V$ in the boundary layer, while the nonviscous fluid reverses the ratio inside the boundary layer of moving vorticity.

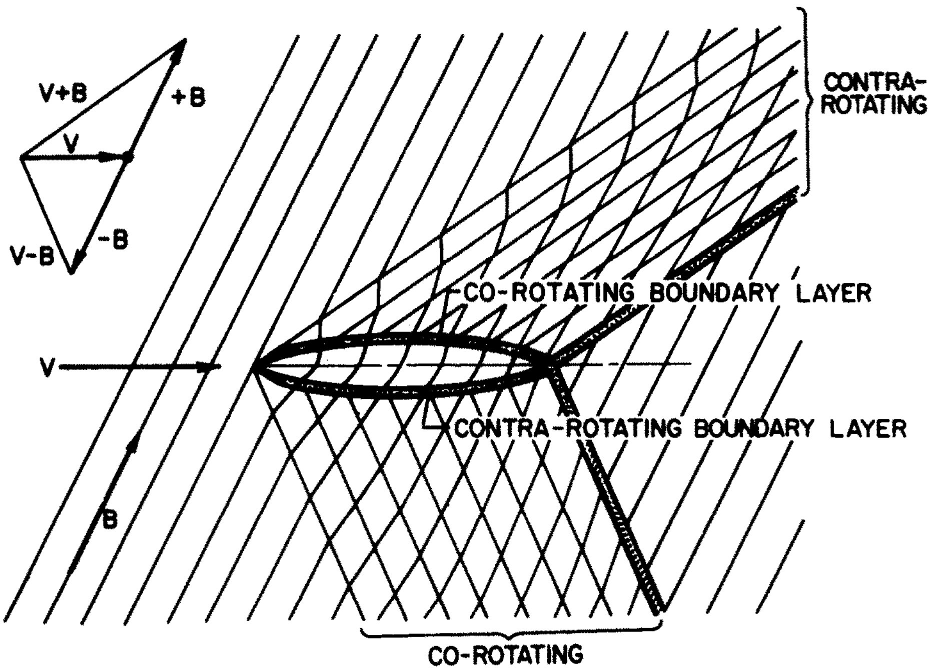

Fig. 5

Historically it was the superconductive fluid having only one boundary-layer solution Q_1 of almost regular behavior on which the reversal of direction at $B = V$ was first discovered, and it was this result that made the classical Kutta-Zhukovskii condition look ridiculous for strong magnetic fields $B > V$ because of the forward wake.

6. Boundary conditions. While the linearization does not guarantee any disclosure of the lift controlling elements, the proper application of the boundary conditions for vanishing imperfections of the fluid on a large variety of examples is still the foundation for any hopes in that direction. The perfect flow itself determines the circulation by the angle of attack between sharp leading and trailing edges only when V and B have different directions. At complete alinement the circulation remains primarily uncontrolled.

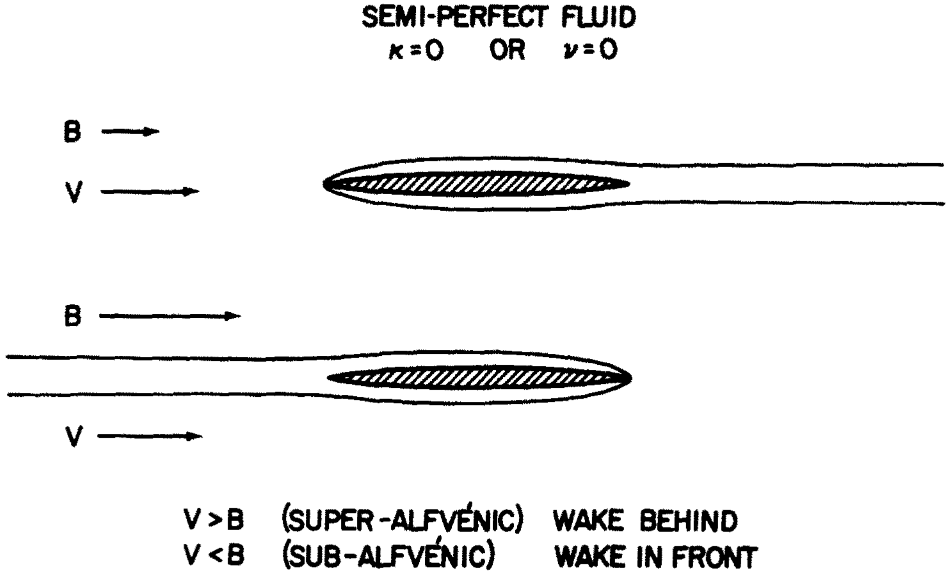

Fig. 6

For the sake of complete confidence in the linearized solutions of the magnetohydrodynamic problem the boundary conditions of the unalined case for the equi-dissipative fluid may be discussed first. Fig.1 shows that the upper and the lower side of the body produce one family of vorticity each already in the case of perfect flow, where the normal but not the tangential velocity at the body contour can be chosen. For being able to prescribe the tangential velocity independently, the second solution of either corotating or contrarotating vorticity is the only help available relying on the completeness of the solutions. Ordinarily the convective velocity of the second wave brands it as an incoming wave with no imformation to carry. But one should never underestimate the power of diffusion. Strong diffusion is able to make its way against any convective speed though, of course, with rather steep decay of the vorticity. If, therefore, the boundary of the body demands the second type of vorticity to be present, a thin layer of the second type is able to exist and it has a tangential velocity according to $V - B_x$ or $V + B_x$, respectively (Fig.5)

One part of the boundary-layer vorticity is able to move in upstream direction when $V - B_x$ is negative. After running off at the trailing or even the leading edge of the body, the second vorticity type finally joins its own family as a narrow band of laces added to the former strip in Fig.4.

Going now back to the case of alined velocity and magnetic fields, the original strips of perfect flow vorticity have folded into the body streamline but the boundary-layer vorticity and the diffusive spread of the edges of the original strips are still available (Fig.6). The equi-dissipative fluid has according to this picture two type of (a) corotating and (b) contrarotating vorticity available with the distinguishable convection speeds $V - B$ and $V + B$. One of these types, the corotating one – if $+V$ and $+B$ are alined – is even able to move toward the leading edge of the body and beyond at sub-Alfvénic speeds.

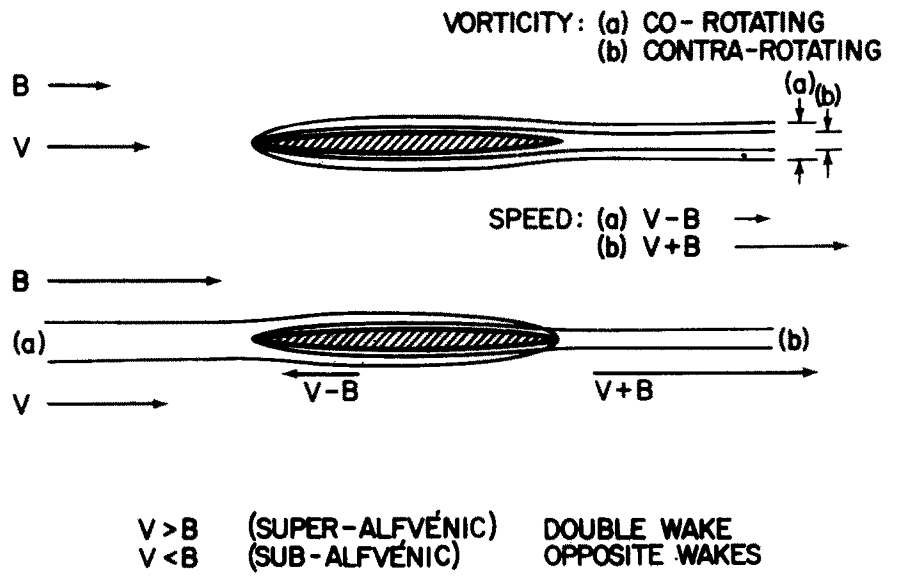

Fig. 7

The semi-perfect fluids have lost one of the diffusive properties that make ordinarily the occurance of discontinuities impossible. Since discontinuities are now acceptable, the double boundary layer of the equi-dissipative fluid changes to a single boundary layer (Fig.7) with the phase velocity $(V^2 - B^2)/V$. The reversal is again at Alfvén speed, but the leading edge is at sub-Alfvénic speeds the only one adjacent to a wake. Nonsteady shedding of votrices may still be expected at the trailing edge, but this process is not available in a factorized differential equation of Q .

7. Conclusion. The two-dimensional magnetohydrodynamic flow past a profile is treated with linearization for only three special ratios between the imperfections caused by viscosity and electrical resistivity zero, one, and infinity. Under these circumstances the Kutta-Zhukovskii problem of lift control is not visible in all its details. If it were, not the generalized Kutta-Zhukovskii conjecture, but its boundary-layer proof would have been presented. In a sense this affair is again in the conjectural state and scientists have still the opportunity to risk their reputation with wrong guesses. Two results are unquestionable: Unalined velocity and magnetic fields have supersonic character, where a sharp leading and a sharp trailing edge control the lift by their angle of attack. The super-Alfvénic speed range of the alined fields has the trailing edge as the lift controlling feature and the trailing edge should be sharp.

The remaining conjecture is about whether the leading and trailing edges should be sharp or not sharp at sub-Alfvénic speeds, where the wake or, at least, half the wake extends forward of the body. Some voices are for a complete reversal of all familiar incompressible or subsonic relations and ask for a sharp leading edge as the means of lift control. One word of caution may be added, that the former suction force on a sharp trailing edge supporting separation has changed to a compression force with a blunting tendency for materials of finite strength.

BIBLIOGRAPHY

1. Sears, W.R. and Resler, E.L. Jr., Theory of Thin Airfoils in Fluids of High Electrical Conductivity. J. Fluid Mech., Vol.5, Part 2, Feb.1959.
2. Resler, E.L. Jr. and McCune, J.E., Electromagnetic Interaction with Aerodynamic Flows. The Magnetohydrodynamics of Conducting Fluids, ed. by Daniel Bershader, Stanford Univ.Press, 1959.
3. Carrier, G.F. and Greenspan, H.P., Magnetohydrodynamic Flow Past a Flat Plate. The Magnetodynamics of Conducting Fluids, ed. by Daniel Bershader, Stanford Univ.Press, 1959 and Fluid Mech., Vol.5, pp.77-96, 1959.
4. Sears, W.R., Studies of an Inviscid Boundary Layer of Magnetohydrodynamics. Tenth Internat.Appl.Mach.Congress, Stresa, Italy, 1960.
5. Busemann, A., On the Kármán Vortex Street in Magnetofluid-Dynamics. 80th Anniversary Proceedings, IAS, 1962.
6. Busemann, A., Relations Between Aerodynamics and Magnetohydrodynamics. NASA SP 25 of University Conference Chicago, November 1-3, 1962.
7. Clauser, F.H., Concept of Field Modes and the Behavior of the Magnetohydrodynamic Field. Phys.of Fluids, Vol.6, pp.231-253, 1963.